Programación estocástica multiobjetivo aplicada al problema de las quemas controladas

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GRUPO ESPAÑOL DE DECISIÓN MULTICRITERIO

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Introduction - prescribed burning

Problem description

Prescribed burning

- Prescribed burning is a fuel reduction technique used to mitigate effects of wildfires.
- For the purpose of fuel management, parks are often divided into treatment units. Scheduling treatments: when? where?

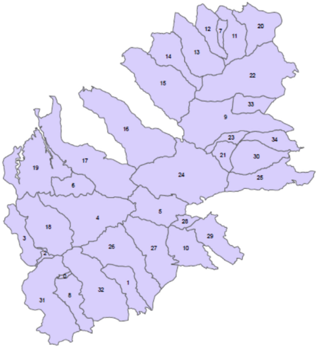

Figure 1: Forest area on the southeast of Victoria

Aftermaths of prescribed burn (Bages, Barcelona)

Introduction - prescribed burning

What is done

Goal

Development of a mathematical programming model to reduce connectivity of high-risk regions (in order to mitigate fire effects), **while** taking into account habitat quality for species living in such area.

Published paper

León, J., Reijnders, V.M.J.J., Hearne, J.W., Ozlen, M. & Reinke, K.J. (2019) 'A Landscape-Scale Optimisation Model to Break the Hazardous Fuel Continuum While Maintaining Habitat Quality' *Environmental Modeling & Assessment* **24**(4), 369–379 doi:10.1007/s10666-018-9642-2.

Introduction - prescribed burning

MIP model description

- Landscape divided in fuel/vegetation uniform units
- Vegetation age sets fuel load and fire risk
  - Age over threshold $\Rightarrow$ high risk unit
  - Adjacent high risk units $\Rightarrow$ high risk connection
  - Number of high risk connections are taken as a proxy for fire risk
- Age also defines suitability for existing species
  - Habitat quality of a unit is defined with a piecewise-linear function of its age
- We formulate a model which decides where to burn each year such that fire risk decreases

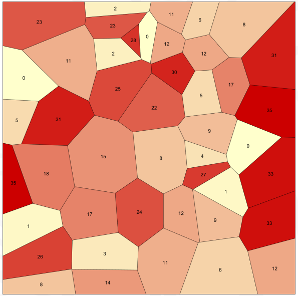

Figure 2: Random landscape, divided in treatment units, each of them with vegetation of certain age

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Methodology - Multiobjective Stochastic Programming

Problem description

Multiobjective Stochastic Programming problem

How can we solve the following?

$$\min_{x \in X} \left(f_1(x,\omega), \ldots, f_k(x,\omega)\right) \qquad \text{(MSP)}$$

Developed tools

- Solution concept for MSP problems, focused on risk aversion
- MP model to obtain such solution

Published paper

León, J., Puerto, J. & Vitoriano, B. (2020)
'A risk-aversion approach for the Multiobjective Stochastic Programming problem'
Mathematics **2020**, 8(11), 2026
doi:10.3390/math8112026

Methodology - Multiobjective Stochastic Programming

Solution concept (informal)

$f_k^j(x)$:

- Minimization
- $x \in X$
- Scenarios:
  - $j = 1, \ldots, J$
  - $\pi_j \geq 0, \sum_j \pi_j = 1$
- Criteria:
  - $k = 1, \ldots, K$
  - $w_k \geq 0, \sum_k w_k = 1$
- Normalized

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0. Set $\beta, r \in (0, 1]$

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Algorithm

0. Set $\beta, r \in (0, 1]$
1. For every $x \in X$ and every criterion define $g_k^\beta(x)$ as:

$$g_k^\beta(x) = \begin{array}{l} \textit{average of worse scenarios for criterion } k \\ \textit{with probabilities adding up to } \beta \end{array}$$

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2. Define $h_r^\beta(x)$ as:

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- Minimization
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3 Search for $x \in X$ minimizing $h_r^\beta(x)$

If we have $f_k^j(x)$ functions to minimize with weight of criteria w_k and probabilities of scenarios π_j inside a domain $x \in X$ with the previously defined solution concept this is equivalent to solving the following:

$$
\begin{aligned}
\min_{z, v_k, z_k, y_{kj}} \quad & z + \sum_k \frac{w_k}{r} v_k \\
\text{s.t.} \quad & z + v_k \geq z_k + \sum_{j=1}^{J} \frac{\pi_j}{\beta} y_{kj} && \forall k \\
& z_k + y_{kj} \geq f_k^j(x) && \forall k, j \\
& y_{kj} \geq 0 && \forall k, j \\
& z_k \text{ free}, v_k \geq 0 && \forall k \\
& z \text{ free}
\end{aligned}
$$

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Revisiting the fuel management problem

- A model was formulated, deciding where to burn each year such that fire risk decreases
- A budget constraint was present, measured in terms of area
- However, often some nominated burns are not completed
- If the budget probability distribution can be estimated $\Rightarrow$ stochastic programming can represent the uncertainty

Paper under (very exhaustive) revision

León, J., Vitoriano, B. & Hearne, J. (202?)
'A risk-averse solution for the prescribed burning problem'
Safety Science

Previous model

The area of the land nominated for burning each year is limited by a known budget (e.g. 5% of total land)

$$\sum_i c_i X_{it} \leq b_t \qquad \forall t$$

MSP applied to prescribed burning

Problem description: budget constraint

Previous model

The area of the land nominated for burning each year is limited by a known budget (e.g. 5% of total land)

Next model

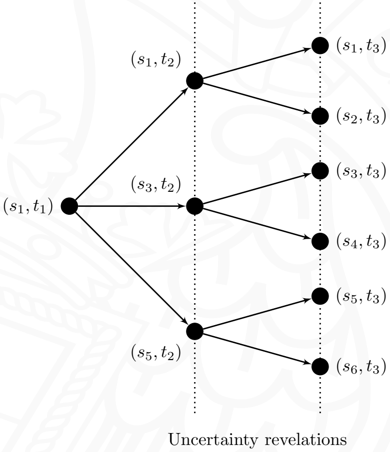
Different budget scenarios may hold (e.g. 3%, 4% and 5% of total land, with given probabilities)

$$\sum_i c_i X_{it} \leq b_t \qquad \forall t$$

$$\sum_i c_i X_{it}^s \leq b_t^s \qquad \forall t, s$$

MSP applied to prescribed burning
Uncertainty tree

Figure 3: Example of scenario tree encoding the uncertainty



Revealing the uncertainty

- Uncertainty can be revealed *within* the burn season
- A sequence for performing the burns in a year is established, and burns are performed until time runs out
- To model this situation, the following constraints have to be included if the budget of scenario s is larger than the one of scenario s':

$$X_{i,t}^{s} \geq X_{i,t}^{s'}$$

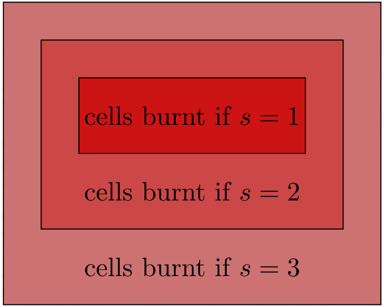


Figure 4: Cells burnt on a given year t, assuming budgets $b^1 < b^2 < b^3$

MSP applied to prescribed burning

Multiple objectives

Objective functions

- Minimize number of high-fuel load connections (proxy of fire-propagation risk)
- Minimize number of high-fuel load units (proxy of fire-ignition risk)
- Maximize habitat quality for two different species

Figure 5: Mallee emu-wren and Hastings river mouse

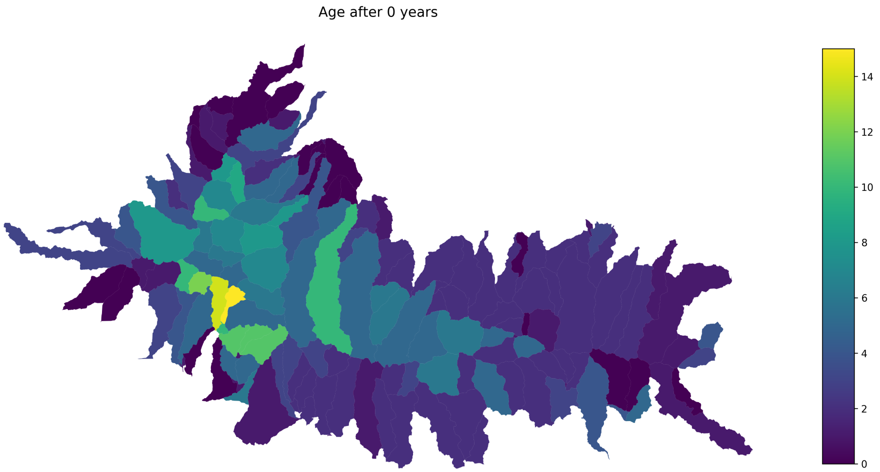
MSP applied to prescribed burning

Case study

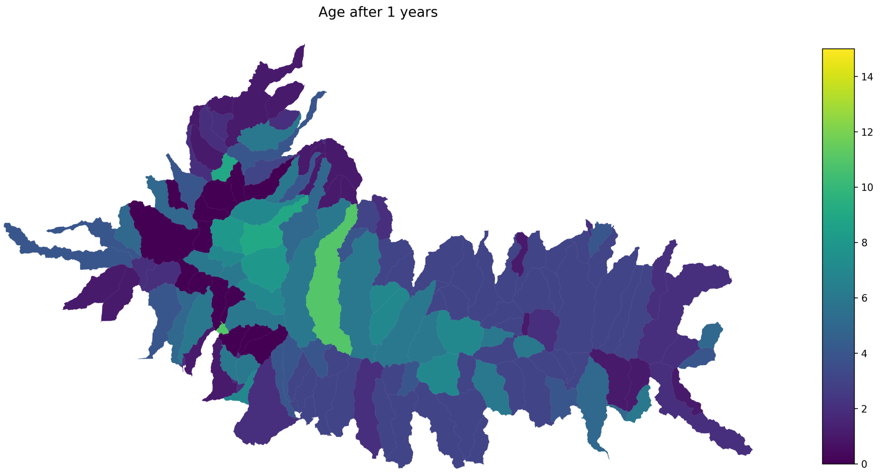
Case study

- Data provided by INFOCA (Andalucía)
- Sierra de Filabres (Almería) and Sierra de Baza (Granada)
- 193 units, 1820km^2
- 20km^2 susceptible to prescribed burning per year
- Including areas with endangered species and private property

MSP applied to prescribed burning



MSP applied to prescribed burning

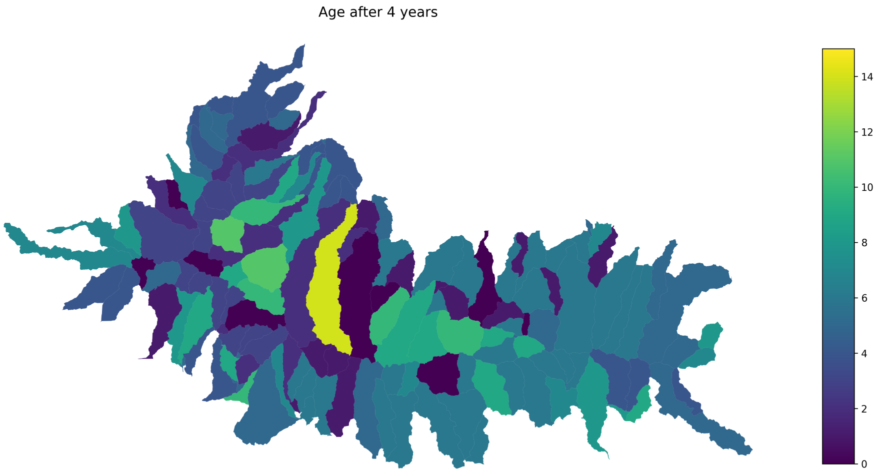


MSP applied to prescribed burning

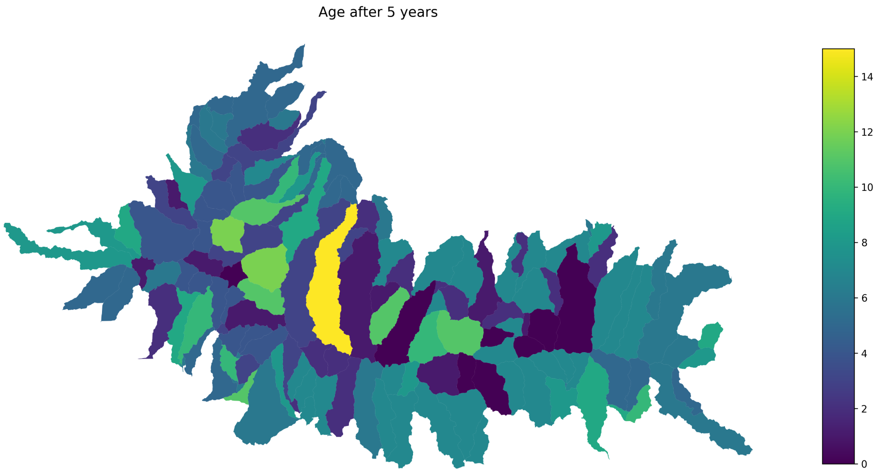
MSP applied to prescribed burning

Age after 3 years

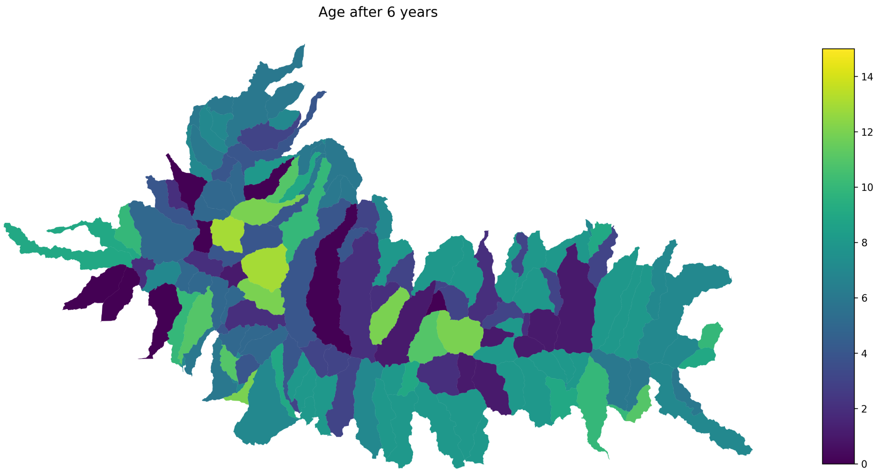
MSP applied to prescribed burning



MSP applied to prescribed burning



MSP applied to prescribed burning



MSP applied to prescribed burning

Age after 7 years

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Conclusions

Conclusions

- A MSP model has been developed and solved using a risk-averse approach, applied to a real case-study in Andalucía
- Results show that using this risk-averse solution instead of an average-average greatly improves the expected outcome of unfavourable conditions, without significantly compromising average outcomes.
- Higher but manageable times for the risk-aversion approach